

CSCI 33500 - Spring 2016 -  
Homework #1, Covering chapters 1 and 2.

Due in class February 18th

All questions are worth the same amount of points.

## 1 Chapter 1 - Mathematical Background

- 1.1 Prove by induction that  $\sum_{i=1}^{N-2} F_i = F_N - 2$   
where  $F_i$  is the  $i$ -th Fibonacci number, as defined in section 1.2 / page 6 of the book.
- 1.2 Prove by induction that  $\sum_{i=1}^N i^3 = (\sum_{i=1}^N i)^2$
- 1.3 Prove that  $2^{99} \equiv 1 \pmod{7}$

## 2 Chapter 2 - Algorithm Analysis

- 2.1 Order the following functions by growth rate:  $N, \sqrt{N}, N^{1.5}, N^2, N \log N, N \log \log N, N(\log N)^2, N \log N^2, 2/N, 2^N, 2^{N/2}, 99$  (constant),  $N^2 \log N, N^3, N^N, N!$ .  
If two functions grow at the same rate, indicate so.
- 2.2 Find two function  $f(N)$  and  $g(N)$  such that neither  $f(N) = O(g(N))$  nor  $g(N) = O(f(N))$ . Explain your answer.
- 2.3 Give a Big-O analysis of the running time of the following code:

```
sum = 0;
for(i=0; i<N; ++i)
    for(j=0; j<i*i; ++j)
        for(k=0; k<j; ++k)
            ++sum;
```

### 3 Extra Credit

Give a Big-O analysis of the running time of the following code:

```
sum = 0;
for(i=0; i<N; ++i)
    for(j=0; j<i*i; ++j)
        if (j%i == 0)
            for(k=0; k<j; ++k)
                ++sum;
```

Compare this to the running time of the algorithm in question 2.3